5. A. P. Alkhimov, A. N. Papyrin, and A. L. Predein, "The use of a high-speed confocal spectrometer in an LDVM system," in: Problems of Gasdynamics [in Russian], Izd. Inst. Teor. Prikl. Mekh. Sibirsk. Otd. Akad. Nauk SSSR, Novosibirsk (1975).
6. B. S. Rinkevichyus and A. V. Tolkachev, "An optical Doppler velocity meter for gas streams," Kvant. Elektron. (Moscow), 1, No. 9 (1974).
7. Yu. G. Vasilenko, V. V. Dontsova, and Yu. N. Dubnitsev, "A laser Doppler velocity meter using a Fabry-Perot interferometer," Avtometriya, No. 3 (1971).
8. D. M. Paul and D. A. Jackson, "Measurement of supersonic velocity and turbulence by laser anemometry," J. Phys. E, 4, 173 (1971).
9. H. Morse, B. Tillis, H. Seifert, and W. Babcock, "Development of laser-Doppler particle sensor for the measurement of velocities in rocket exhausts," J. Spacecr. Rockets, 6, No. 3, 1168 (1969).
10. H. G. Heard, Laser Parameter Measurement Handbook, Wiley, New York (1968).
11. B. S. Rinkevichyus, "The Doppler method of measuring local velocities using lasers," Usp. Fiz. Nauk, 111, No. 2, 305 (1973).
12. A. P. Alkhimov, A. N. Papyrin, A. L. Predein, and R. I. Soloukhin, "An experimental study of the effect of the velocity lag of fine particles in a supersonic gas stream," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1977).

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# ROLE OF VIBRATIONAL RELAXATION IN THE NONEQUILIBRIUM FLOW OF AIR IN NOZZLES

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As is known, the nonequilibrium excitation of vibrational degrees of freedom occurs along with chemical reactions in a high-enthalpy air stream in a nozzle. The role of vibrational nonequilibrium has been studied insufficiently, and in calculations it is assumed, as a rule, that the vibrational degrees of freedom are excited in equilibrium [1-4].

The nonequilibrium air flow in a nozzle of hyperbolic profile is analyzed in the present report for the ranges of temperatures and stagnation pressures of $3000 \le T_0' \le 5000$°K and $1 \le p_0' \le 100$ atm characteristic of the existing hypersonic experimental installations. The dependence of the frozen-in internal energy on the mode of flow is analyzed on the basis of the calculations conducted. Conclusions are drawn about the influence of vibrational relaxation on the gasdynamic parameters of a stream.

## Gas-Kinetic Model

The following system of chemical reactions [4] is taken as basic for air in the range of temperatures and pressures under consideration:

$$\begin{gathered} O_2 + M \rightleftarrows 2O + M,\ N_2 + M \rightleftarrows 2N + M, \\ NO + M \rightleftarrows N + O + M,\ O + N_2 \rightleftarrows NO + N, \\ NO + O_2 \rightleftarrows N + O_2,\ N_2 + O_2 \rightleftarrows 2NO, \end{gathered}$$

where M is any of the molecules $O_2$, $N_2$, O, NO, or N.

It is assumed that the vibrational temperature of nitric oxide is in equilibrium with the translational temperature. The kinetic equations of [3] describing the vibrational relaxation in a mixture of polyatomic gases were used to calculate the vibrational energy of the $N_2$ and $O_2$ molecules.* The system of one-dimensional gasdynamic equations is described in detail in [1-5]. The expressions for the vibrational relaxation times and the reaction rate constants are taken from [6-8].

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*As calculations show, in this case the influence of dissociation on the excitation of the vibrational degrees of freedom of the molecules is slight.

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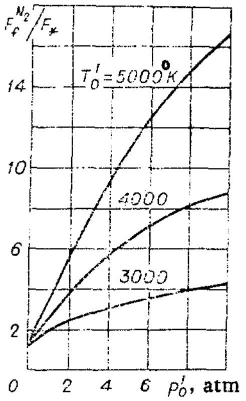

Fig. 1

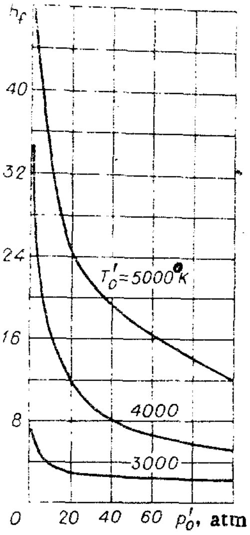

Fig. 2

The calculations were performed for axisymmetric hyperbolic nozzles, whose cross-sectional area $F'$ is described by the equation $F'/F_*' = 1 + (x'/l')^2$, where $l' = r_*'/\tan\varphi'$ ($\varphi'$ is the half-angle of the asymptotic cone, $x'$ is the coordinate along the nozzle axis, $F_*' = \pi r_*'^2$, $r_*'$ is the radius of the minimum cross section of the nozzle, and a prime denotes a dimensional physical quantity). In the subsonic part of the nozzle $l' = 0.262$ cm ($\varphi_1' = 45°$ and $r_*' = 0.262$ cm), while in the supersonic part $l' = 1$ cm. The calculation method is presented in detail in [5, 9, 10].

## Results of Calculation

The specific internal energy frozen into the stream is $w_f' = e_f' + h_f'$, where $e_f'$ is the frozen-in specific vibrational energy and $h_f'$ is the frozen-in specific energy due to dissociation.

The calculations performed showed that the $N_2$ molecules make the main contribution to the frozen-in vibrational energy $e_f'$ for the modes of flow under consideration, since the frozen-in concentration of $N_2$ is considerably higher than the frozen-in concentrations of $O_2$ and NO. The energy contribution of the NO molecules to $e_f'$ is slight. The energy contribution of the $O_2$ molecules is about 10% of the total frozen-in vibrational energy.

The cross sections $F_f^{N_2}/F_*$ in which the concentration and vibrational energy of the $N_2$ molecules are frozen in are presented in Fig. 1 as functions of the stagnation pressure $p_0'$ at several values of the stagnation temperature $T_0'$. The cross section for the freezing in the internal energy of the $N_2$ molecules formally corresponds to the vibrational energy of $N_2$, which differs from its frozen-in value by 1%. The freezing in of the concentrations of the air components, with the exception of the concentration of nitrogen atoms which is negligibly small for the range of stagnation parameters under consideration, occurs earlier than the freezing in the $N_2$ vibrations and considerably earlier than the freezing in of the $O_2$ vibrations. The dimensionless frozen-in chemical energy $h_f = h_f'/R'T_\infty'$ ($R'$ is the universal gas constant and $T_\infty' = 273°K$) as functions of $p_0'$ for several values of $T_0'$ is presented in Fig. 2.

The dependence of the ratio $w_f/w_0$ of the frozen-in internal energy to the total energy of the stream on the stagnation pressure for different $T_0'$ is presented in Fig. 3. At $4000 \leq T_0' \leq 5000°K$ and $p_0' = 1$ atm the frozen-in energy can comprise more than 40% of the total energy of the stream.

The contribution of the frozen-in vibrational energy to the total frozen-in energy of the stream as a function of the stagnation parameters is reflected in Fig. 4. With an increase in pressure the vibrational relaxation begins to play an ever more important role, and at $T_0' = 3000°K$ the contribution of the vibrations to the frozen-in energy of the stream is comparable with the contribution of the chemical reactions. With an increase in temperature the role of the vibrations becomes ever less important, and at $T_0' = 5000°K$ and an arbitrary $p_0'$ from the pressure range under consideration the frozen-in vibrational energy comprises less than 25% of the frozen-in internal energy of nonequilibrium air. This is connected with the fact that in a chemically relaxing stream the concentrations of the atomic components begin to grow and the concentrations of molecules decline accordingly.

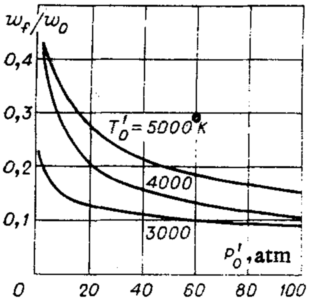

Fig. 3

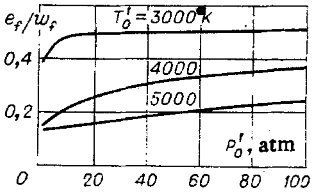

Fig. 4

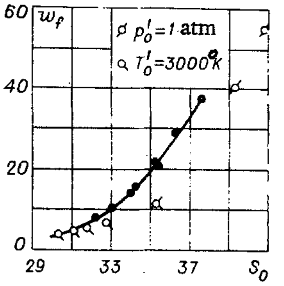

Fig. 5

## A Comment on the Entropy Correlation

The so-called entropy correlation represents the dependence of the frozen-in parameters of an air stream on the entropy in the forechamber [1, 11]. It is very convenient for the quantitative determination of the concentrations and gasdynamic parameters of a nonequilibrium air stream in a nozzle. The entropy correlation between the chemically frozen-in energy and the concentrations of the neutral components of air on the basis of an analysis of the calculated data of a number of authors are presented in [12] for a wide range of stagnation parameters $4000 \le T_0' \le 15{,}000°K$ and $10 \le p_0' \le 8957$ atm for the parameter $l' = 1$ cm ($\varphi_1' = \varphi_2'$).

It is interesting to point out the region of application of the entropy correlation. The dependence of the dimensionless frozen-in internal energy $w_f = w_f'/R'T_\infty'$ on the entropy $S_0$ in the forechamber ($S_0 = S_0'\mu_\infty'/R'$, where $\mu_\infty' = 29$ g/mole) obtained in the present work is plotted in Fig. 5.

The correlation is fully satisfactory for values of $S_0'$ corresponding to stagnation parameters $p_0' \ge 10$ atm and $T_0' \ge 4000°K$. In the region of smaller values of $S_0'$ the limitation on $p_0'$ becomes stricter with a decrease in $T_0'$. At $T_0' = 3000°K$, in particular, the entropy correlation is valid for $p_0' \gtrsim 25$ atm. Refinement of the kinetic model of air in the region of low stagnation pressures might permit an expansion of the region of application of the entropy correlation.

## LITERATURE CITED

1. V. P. Agafonov, V. K. Vertushkin, A. A. Gladkov, and O. Yu. Polyanskii, Nonequilibrium Physicochemical Processes in Aerodynamics [in Russian], Mashinostroenie, Moscow (1972).
2. V. V. Lunev, Hypersonic Aerodynamics [in Russian], Mashinostroenie, Moscow (1975).
3. E. V. Stupochenko, S. A. Losev, and A. I. Osipov, Relaxation Processes in Shock Waves [in Russian], Nauka, Moscow (1965).
4. Ya. B. Zel'dovich and Yu. P. Raizer, The Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, Academic Press.
5. M. F. Budarina, V. N. Komarov, and G. N. Sayapin, "Calculation of nonequilibrium air flows in nozzles," Tr. Tsentr. Aerogidrodinam. Inst., No. 1701 (1975).
6. J. J. Martin, Atmospheric Reentry, Prentice-Hall, Englewood Cliffs, New Jersey (1966).
7. B. F. Gordiets, I. S. Mamedov, A. I. Osipov, and L. A. Shelepin, "Distribution of vibrational energy in gas mixtures," Preprint No. 31, Fiz. Inst. Akad. Nauk SSSR, Moscow (1972).
8. F. R. Gilmore, E. Bauer, and J. W. McGowan, "A review of atomic and molecular excitation mechanisms in nonequilibrium gases up to 20,000°K," J. Quant. Spectrosc. Radiat. Transfer., 9 (1969).
9. A. Q. Eschenroeder, D. W. Boyer, and J. G. Hall, "Nonequilibrium expansions of air with coupled chemical reactions," Phys. Fluids, 5, No. 5, 615 (1962).
10. V. P. Stulov and V. P. Shkadova, "On one-dimensional nonequilibrium air flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1968).
11. K. N. C. Bray, "Simplified sudden-freezing analysis for nonequilibrium nozzle flows," ARS J., 31, No. 6 (1961).
12. C. J. Harris, "Comment on nonequilibrium effects on high-enthalpy expansion of air," AIAA J., 4, No. 6 (1966).